MACHINE DESIGN

Simple Stresses in Machine Parts

Introduction

- In engineering practice, the machine parts are subjected to various forces which may be due to either one or more of the following:

1. Energy transmitted,
2. Weight of machine,
3. Frictional resistances,
4. Inertia of reciprocating parts,
5. Change of temperature, and
6. Lack of balance of moving parts.

Load

The defined as any external force acting upon a machine part.

Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as unit stress orsimply a stress.

$$\text{Stress, } \sigma = P/A$$

where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as unit strain or simply a strain.

Strain, $\varepsilon = \delta l / l$ or $\delta l = \varepsilon . l$

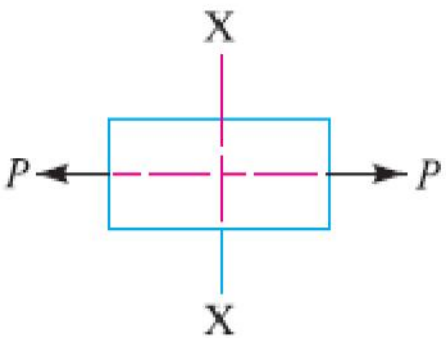
where

δl = Change in length of the body, and

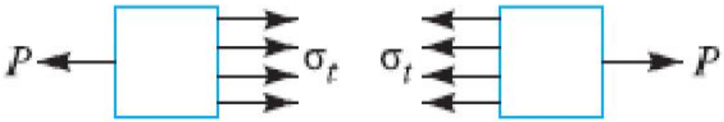
l = Original length of the body.

Tensile Stress and Strain

When a body is subjected to two equal and opposite axial pulls P (also called **tensile load**) as shown in Fig. (a), then the stress induced at any section of the body is known as **tensile stress** as shown in Fig.(b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as **tensile strain**.



(a)



(b)

Let P = Axial tensile force acting on the body,

A = Cross-sectional area of the body,

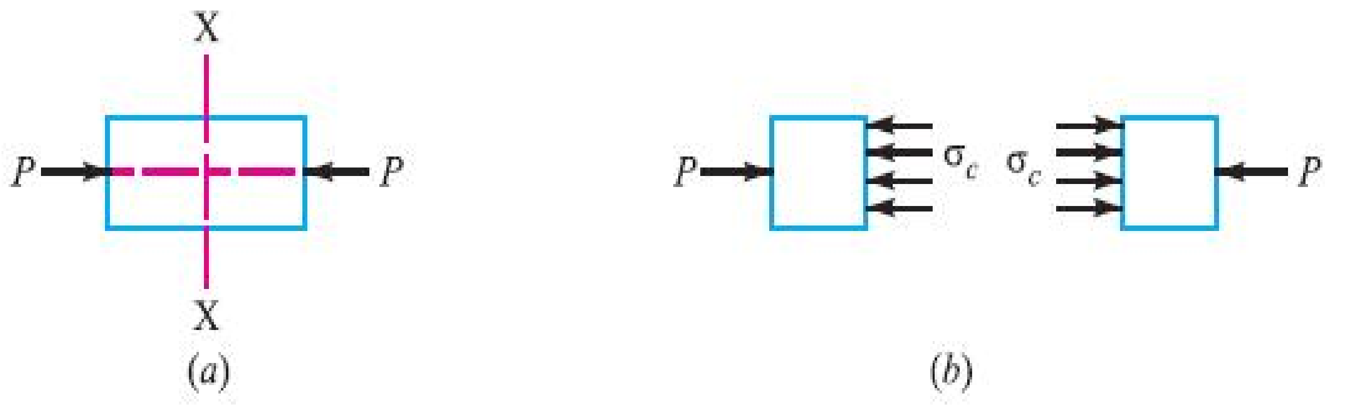
l = Original length, and

δl = Increase in length.

$\therefore$ Tensile stress, $\sigma_t = P/A$

and tensile strain, $\varepsilon_t = \delta l / l$

Compressive Stress and Strain



Let P = Axial compressive force acting on the body,

A = Cross-sectional area of the body,

l = Original length, and

δl = Decrease in length.

∴ Compressive stress, $\sigma_c = P/A$

and compressive strain, $\varepsilon_c = \delta l / l$

Young's Modulus or Modulus of Elasticity

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain, i.e.

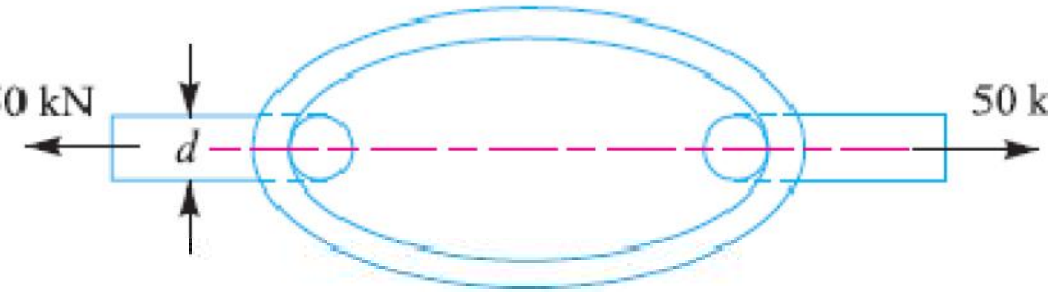
$$\sigma \propto \varepsilon \quad \text{or} \quad \sigma = E.\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where E is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity***. In S.I. units, it is usually expressed in GPa *i.e.* GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

Example(1): A coil chain of a crane required to carry a max. load of 50 kN, is shown in Figure.

Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.



Solution. Given : $P - 50$ kN $- 50 \times 10^3$ N ; $\sigma_t - 75$ MPa $- 75$ N/mm^2

Let d = Diameter of the link stock in mm.

$\therefore$ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$

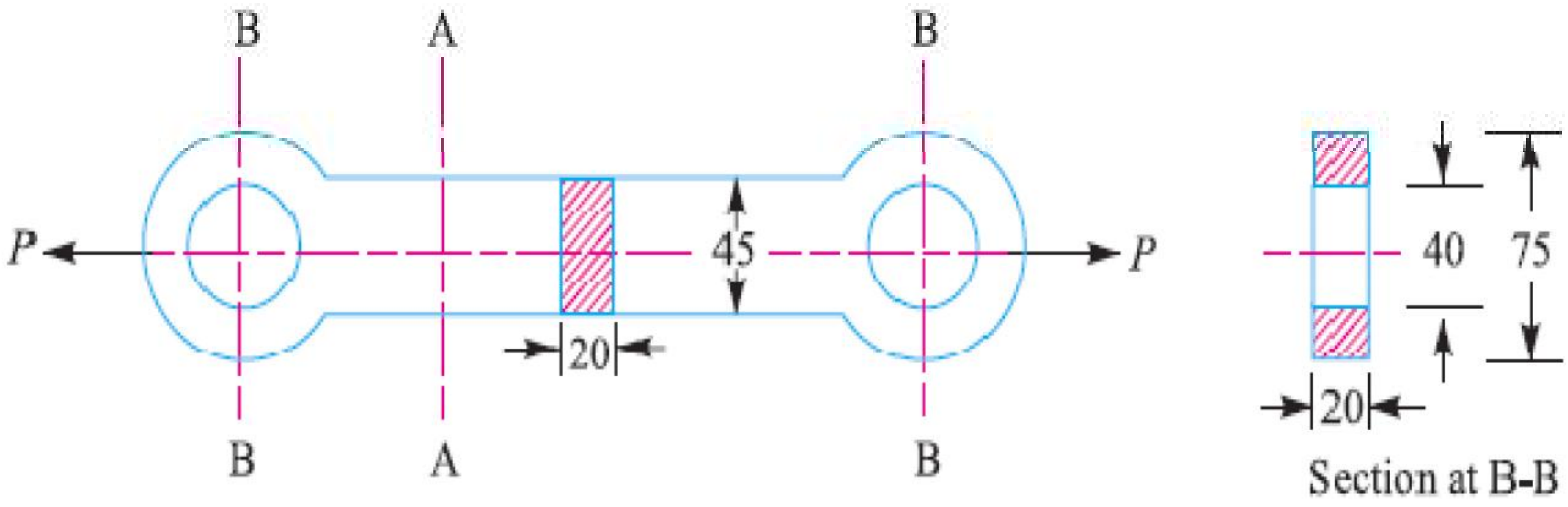
We know that the maximum load (P),

$$50 \times 10^3 = \sigma_t . A = 75 \times 0.7854\, d^2 = 58.9\, d^2$$

$\therefore$ $d^2 - 50 \times 10^3 / 58.9 - 850$ or $d - 29.13$ say 30 mm

Example(2): A cast iron link, as shown in Figure, is required to transmit a steady tensile load of 45 kN.

Find the tensile stress induced in the link material at sections A-A and B-B.



Solution. Given : $P = 45 \text{ kN} = 45 \times 10^3 \text{ N}$

Tensile stress induced at section A-A

We know that the cross-sectional area of link at section *A-A*,

$$A_1 = 45 \times 20 = 900 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section *A-A*,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{900} = 50 \text{ N/mm}^2 = 50 \text{ MPa} \quad \textbf{Ans.}$$

Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

$$A_2 = 20\,(75 - 40) = 700 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section *B-B,*

$$\sigma_{t2} = \frac{P}{A_2} = \frac{45 \times 10^3}{700} = 64.3 \text{ N/mm}^2 = 64.3 \text{ MPa} \quad \textbf{Ans.}$$

Shear Stress and Strain

When a body is subjected to **two equal** and **opposite forces** acting **tangentially** across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called **shear stress**.
The corresponding strain is known as **shear strain** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by tau (|) and phi (φ) respectively. Mathematically,

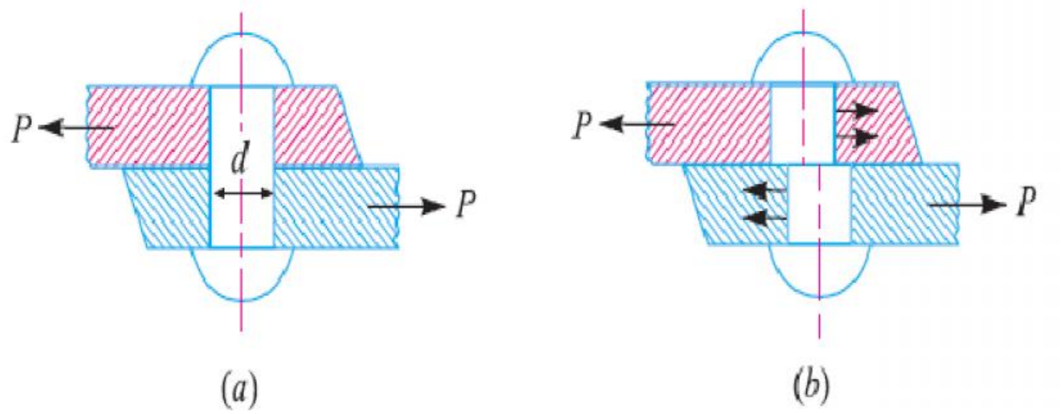
$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

A body consisting of two plates connected by a rivet as shown in Fig.(a). The tangential force P tends to shear off the rivet at one cross-section as shown in Fig.(b). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at **one cross-section** of the rivet), the rivets are said to be in **single shear**. In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

and shear stress on the rivet cross-section,

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

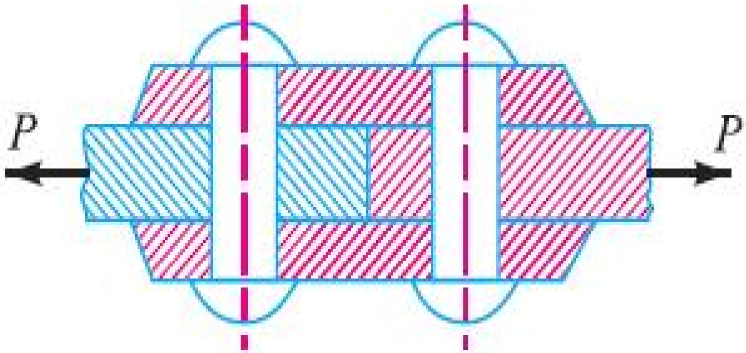


when the shearing takes place at **two cross-sections** of the rivet), then the rivets are said to be in **double shear**. In such a case, the area resisting the shear off the rivet,

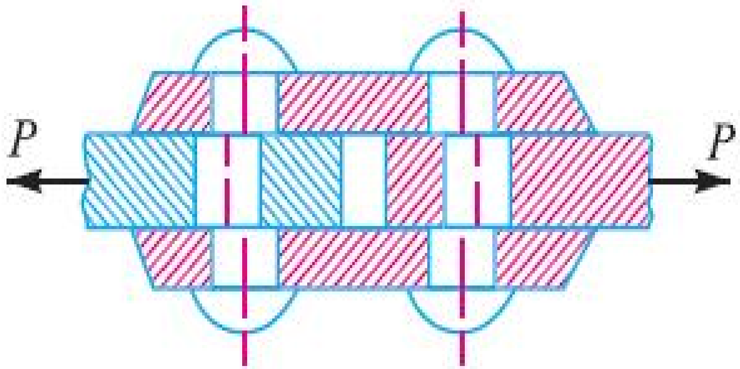
$$A = 2 \times \frac{\pi}{4} \times d^2 \quad \text{... (For double shear)}$$

and shear stress on the rivet cross-section,

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$



(a)



(b)

Punched or drilled the metal plates, tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter ‘d’ is to be punched in a metal plate of thickness ‘t’, then the area to be sheared,

$$A = \pi d \times t$$

and the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi d \times t \times \tau_u$$

where τ_u = Ultimate shear strength of the material of the plate.

Example(3): Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm^2.

Solution. Given: $d = 60$ mm ; $t = 5$ mm ; $\tau_u = 350$ N/mm^2

We know that area under shear,

$$A = \pi d \times t = \pi \times 60 \times 5 = 942.6 \text{ mm}^2$$

and force required to punch a hole,

$$P = A \times \tau_u = 942.6 \times 350 = 329\,910 \text{ N} = 329.91 \text{ kN}$$ **Ans.**

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \cdot \phi \quad \text{or} \quad \tau / \phi = C$$

where
τ = Shear stress,
ϕ = Shear strain, and
C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

Material	*Modulus of rigidity (C) in GPa i.e. GN/m^2 or kN/mm^2*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10