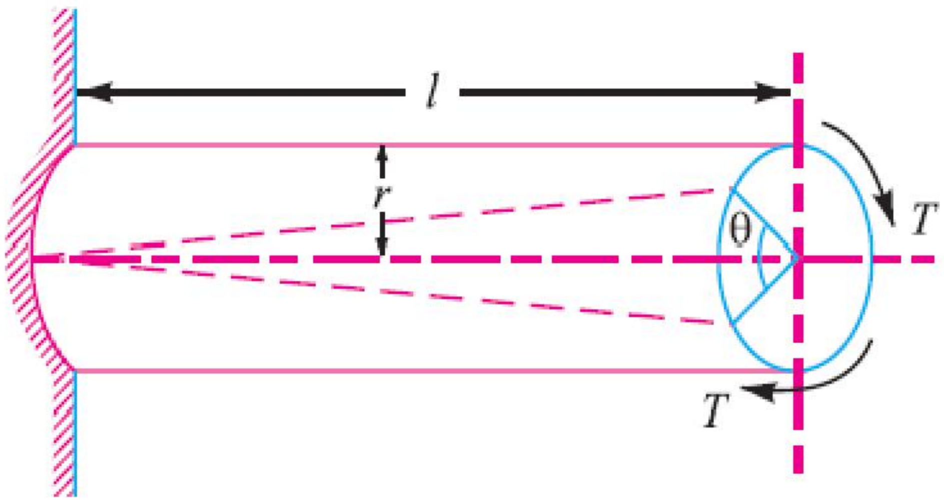
MACHINE DESIGN

Torsional and Bending Stresses in Machine Parts

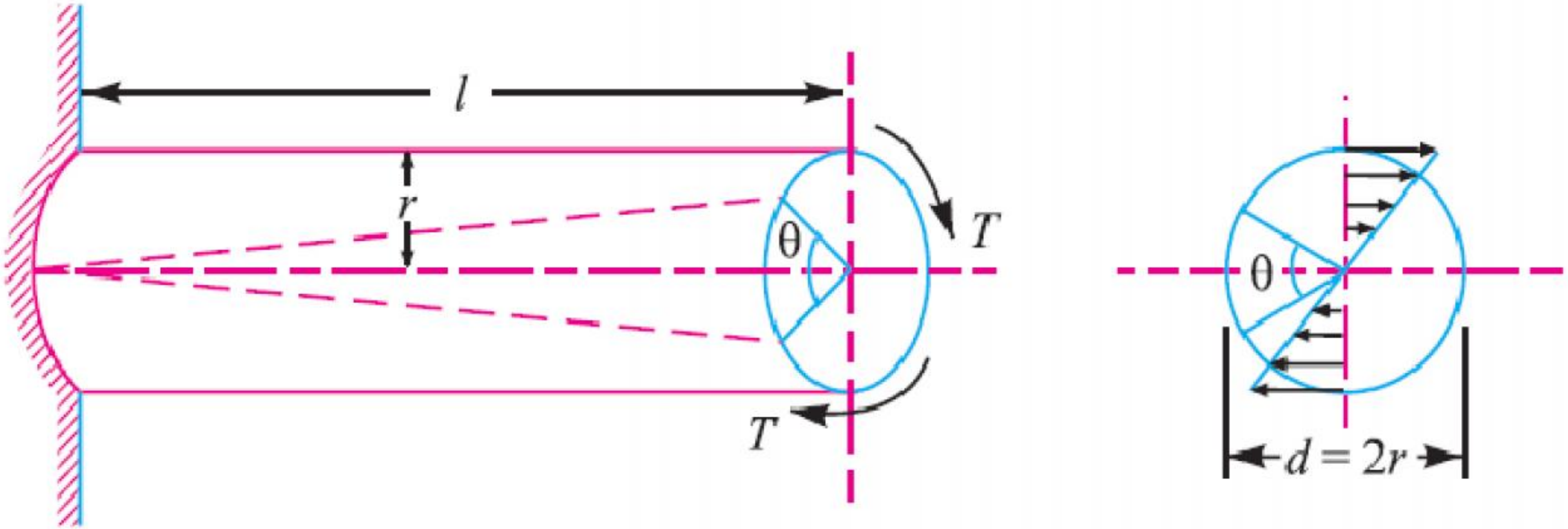
Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress. It is zero at the centroidal axis and maximum at the outer surface.



Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Figure. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\boxed{\frac{\tau}{r} = \frac{T}{J} = \frac{C \, . \, \theta}{l}}$$



$$\frac{\tau}{r} = \frac{T}{J} = \frac{C\,.\,\theta}{l}$$

τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

In case of a hollow shaft with external diameter (do) and internal diameter (di), the polar moment of inertia,

$$J = \frac{\pi}{32} \left[(d_o)^4 - (d_i)^4 \right]$$

to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$\boxed{P = \frac{2\,\pi\,N\,.\,T}{60} = T\,.\,\omega}$$

where T – Torque transmitted in N-m, and

ω = Angular speed in rad/s.

$$\left(\because\ \omega = \frac{2\,\pi\,N}{60}\right)$$

Example(1): A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa. a steady tensile load of 45 kN.

Find the tensile stress induced in the link material at sections A-A and B-B.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$\therefore$ $T_{mean} = 100 \times 10^3/16.76 = 5966.6$ N-m

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

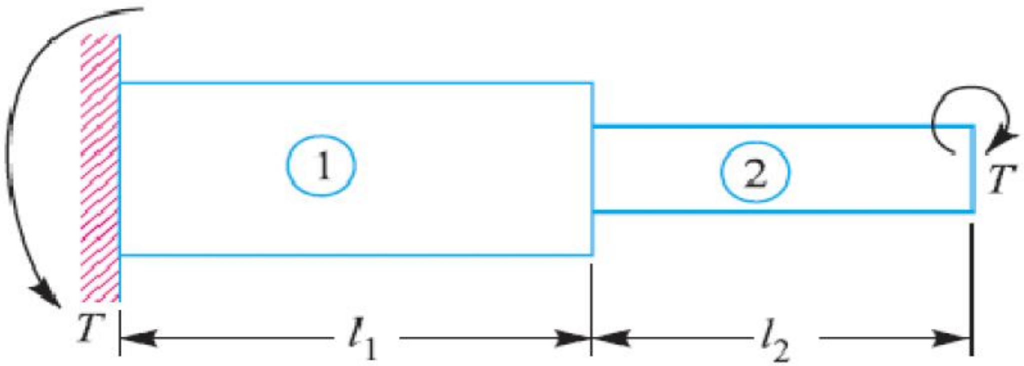
$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3 / 13.75 = 542.4 \times 10^3 \quad \text{or} \quad d = 81.5 \text{ mm}$$

Shafts in Series and Parallel

1- Shaft in Series

When two shafts of different diameters are connected together to form one shaft, it is then known as composite shaft. If the driving torque is applied at one end and the resisting torque at the other end, then the shafts are said to be connected in series as shown in Figure. In such cases, each shaft transmits the same torque and the total angle of twist is equal to the sum of the angle of twists of the two shafts.



1- Shaft in Series (Cont.)

Mathematically, total angle of twist,

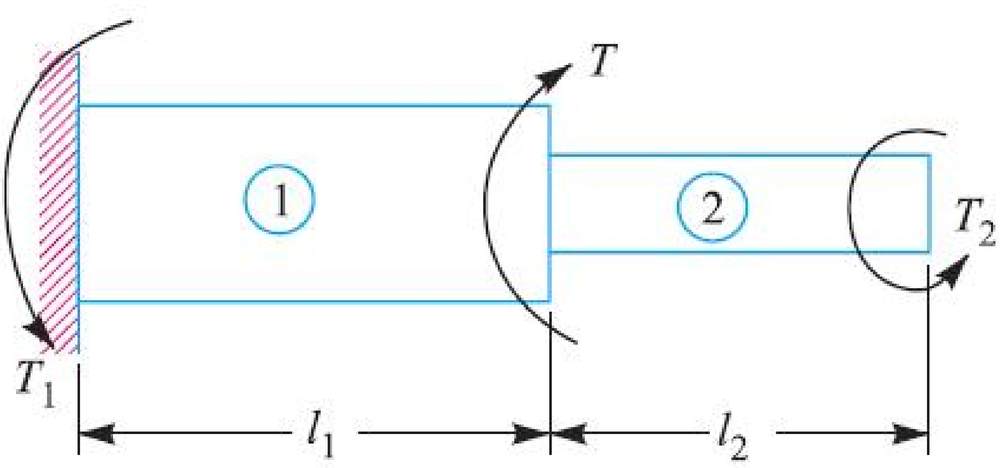
$$\theta - \theta_1 + \theta_2 - \frac{T.l_1}{C_1 J_1} + \frac{T.l_2}{C_2 J_2}$$

If the shafts are made of the same material, then $C_1 = C_2 = C$.

$$\therefore \qquad 0 = \frac{T.l_1}{C.J_1} + \frac{T.l_2}{C.J_2} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$

2- Shaft in Parallel

When the driving torque (T) is applied at the junction of the two shafts, and the resisting torques T1 and T2 at the other ends of the shafts, then the shafts are said to be connected in parallel, as shown in Figure. In such cases, the angle of twist is same for both the shafts,



2- Shaft in Parallel (Cont.)

$$\theta_1 = \theta_2$$

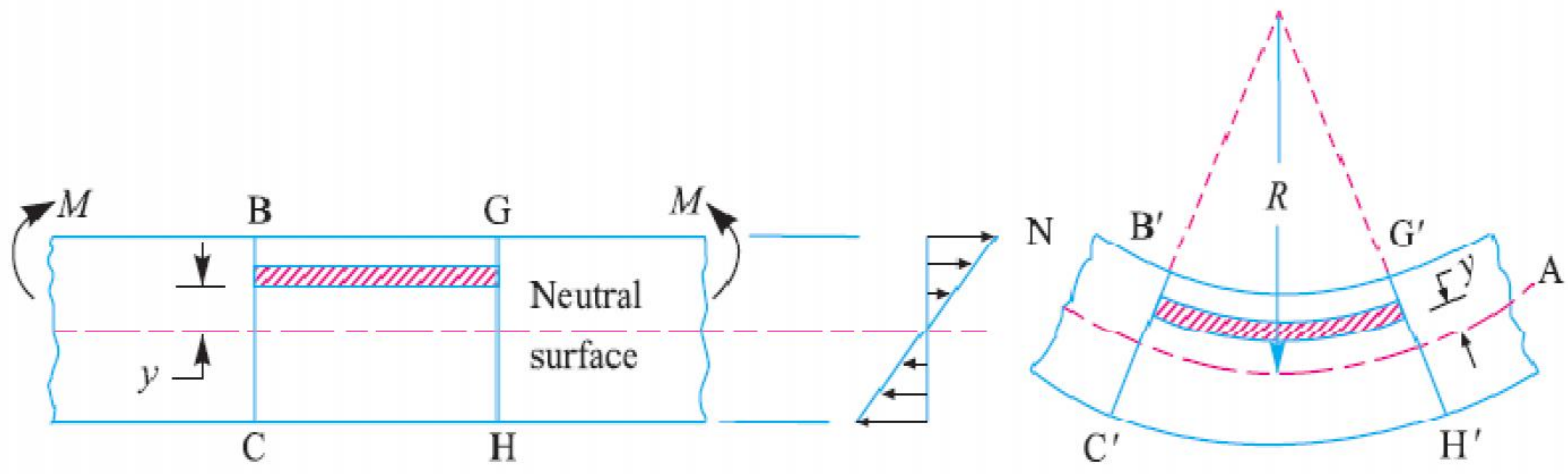
$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

$$T = T_1 + T_2$$

If the shafts are made of the same material, then $C_1 = C_2$.

$$\therefore \qquad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}$$

Bending Stress in Straight Beams



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

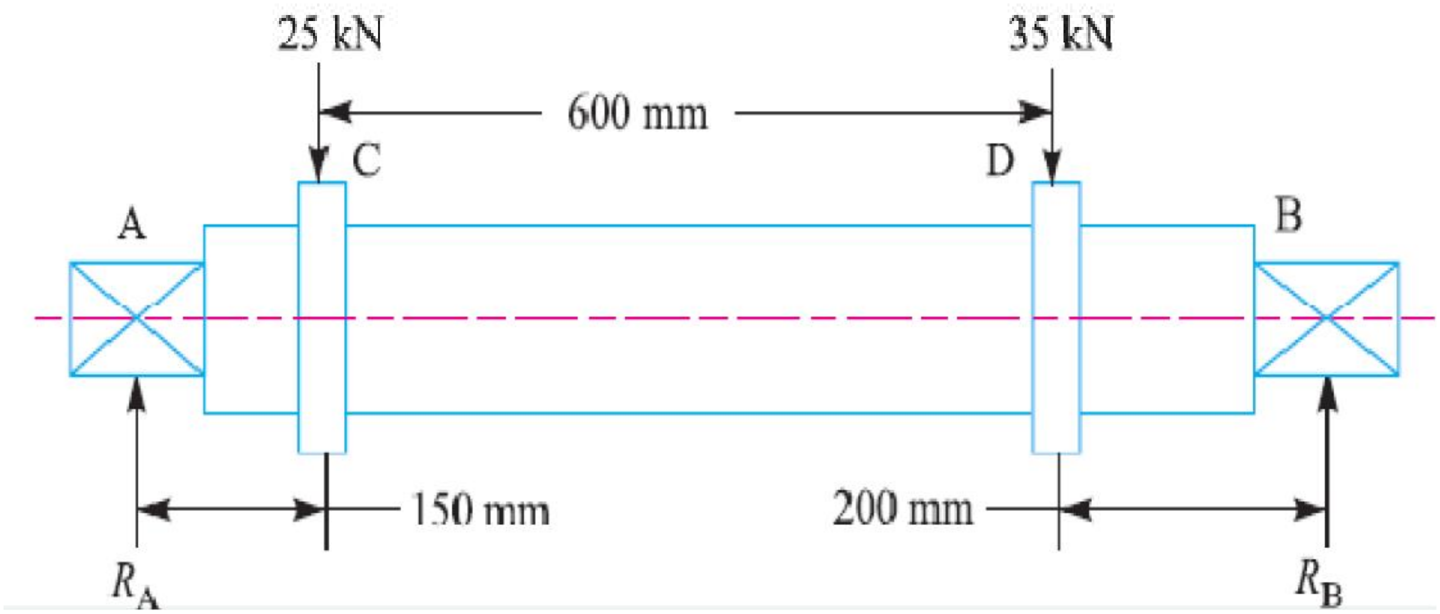
The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes : 1. The neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or Tsection, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Example(2): A pump lever rocking shaft is shown in Figure. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.



Solution. Given : $\sigma_b = 100 \text{ MPa} = 100 \text{ N/mm}^2$

Let R_A and R_B = Reactions at A and B respectively.

Taking moments about A, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\,000$$

$$\therefore \quad R_B = 30\,000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and $$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

$\therefore$ Bending moment at C

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6 \text{ N-mm}$

We see that the maximum bending moment is at D, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3$$

$$= 0.0982\, d^3$$

We know that bending stress (σ_b),

$$100 = \frac{M}{Z}$$

$$= \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3$ or $d = 86.3$ say 90 mm